

1 **Temperature-dependent droplet impact dynamics of a water droplet on**
2 **hydrophobic and superhydrophobic surfaces: an experimental and**
3 **predictive machine learning–based study**

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11

12 **Abstract**

13 Heightening the water repellency of surfaces can serve anti-icing purposes by removing
14 water drops before they freeze and adhere to a surface. Here we study the impact dynamics
15 of water droplets on silicone rubber surfaces—ranging from hydrophobic to
16 superhydrophobic—at -20 , -10 , and 25 °C. We evaluate the influence of static contact
17 angle, contact angle hysteresis, surface roughness, temperature, impacting velocity, and
18 droplet diameter on droplet behavior (e.g., deposition, bouncing, splash). **Minor effect of
19 temperature on droplet dynamics on microstructured surfaces for a wide range of Weber
20 and Reynolds numbers is observed. Experimental observations show that full bouncing
21 only occurs on superhydrophobic surfaces with a $CA > 160^\circ$ and a $CAH < 2^\circ$ at
22 temperatures above 0 °C for $We < 110$ and $Re < 5000$. Increasing the impact velocity of the
23 droplet on rough surfaces heightens the probability of splashing.** This experimental data is
24 then coupled with machine-learning techniques (logistic regression, decision tree, and
25 random forest) to comprehensively investigate droplet impact behavior on hydrophobic
26 and superhydrophobic surfaces at various temperatures. We predict the behavior
27 probability of impacting droplets on surfaces as a function of Weber number, Reynolds
28 number and surface features (static contact angle, contact angle hysteresis, temperature,
29 and surface roughness). Our experimental results and machine learning–based predictions
30 are highly consistent, demonstrating that machine learning can effectively predict droplet
31 motion on hydrophobic and superhydrophobic silicone rubber surfaces at different
32 temperatures.

33 **Keywords:** superhydrophobic, surface features, temperature, droplet impact, freezing,
34 machine learning

35 **1. Introduction**

36 Ice formation on structures poses a major hazard for exposed infrastructure and equipment
37 and can lead to serious incidents, including aircraft crashes, the collapse of transmission
38 lines, and damage to industrial facilities [1–4]. Ice removal techniques from solid surfaces
39 can be classified into active de-icing and passive anti-icing methods. The latter means
40 offers numerous advantages over active de-icing [5–7]. Passive anti-icing includes
41 icephobic surfaces, which prevent ice formation on the surface without requiring external
42 energy. These icephobic surfaces usually operate by exhibiting an improved water
43 repellency (droplet mobility) to remove water droplets before their freezing, hinder ice
44 nucleation on the surface, reduce the ice adhesion force, or a combination of these
45 properties [8–11].

46 Improving our knowledge about ice nucleation, ice formation, and ice adhesion processes
47 can help overcome issues related to surface icing and enhance the design of icephobic
48 surfaces. Superhydrophobic surfaces can be very effective in preventing ice formation
49 compared with hydrophilic or hydrophobic surfaces because of their ability to repel
50 impacting droplets before ice nucleation [12,13]. Nonetheless, some studies have reported
51 the opposite effect of superhydrophobicity on ice mitigation [14,15].

52 Much focus has been placed on ice accretion and the related impact and freezing processes
53 of water droplets on cold surfaces [16–19]. The impinging of a droplet on a surface leads
54 to a conversion from inertial energy to surface energy, droplet spreading, and droplet

55 deformation. The wetting properties of surfaces and the extent of energy dissipation when
56 a droplet lands on a surface produce various impact droplet patterns [20]. For
57 superhydrophobic surfaces, the pure conversion of kinetic energy into surface energy is
58 expected because the air pockets trapped at the interface minimize the dissipation of the
59 kinetic energy of the droplet. Thus, bouncing is possible because of the sufficient energy
60 stored in deformation during droplet impact. However, high-energy dissipation for other
61 surfaces, such as hydrophilic surfaces, reduces the kinetic energy available for bouncing
62 [21,22].

63 Different phenomena arise during droplet impaction onto a solid surface, such as
64 deposition, receding, splashing, and bouncing [23]. The outcome of an impacting droplet
65 is determined by multiple factors, including droplet properties (e.g., viscosity, density,
66 surface tension), operational parameters (e.g., velocity), and surface characteristics (e.g.,
67 wettability) that are altered by surface roughness or texturing [24–26]. Dimensionless
68 parameters are used to account for these factors affecting droplet impact dynamics. These
69 critical parameters include the Reynolds number, $Re = \rho u_0 d_0 / \mu$, the Weber number,
70 $We = \rho u_0^2 d_0 / \sigma$, the capillary number, $Ca = \mu u_0 / \sigma$, and the Ohnesorge number, $Oh =$
71 $\mu / \sqrt{\rho \sigma d_0}$, where u_0 is the impact velocity, d_0 is the initial droplet diameter, and ρ , μ , and
72 σ are the liquid density, viscosity, and surface tension, respectively [27]. Surface
73 wettability is commonly stated in terms of the contact angle (CA) of a water droplet and
74 contact angle hysteresis (CAH), which is the difference between the advancing and
75 receding CAs. The latter is often used as a measure of droplet mobility across a surface.

76 A critical characteristic affecting liquid droplets on cold solid substrates is surface
77 temperature. Under icing conditions, water droplet properties, wetting properties, and frost
78 formation depend greatly on surface temperature and cause the impacted water droplet to
79 move less rapidly across the cold surface [28]. The effect of substrate roughness and
80 temperature on droplet impact dynamics on cooled superhydrophobic surfaces has been
81 discussed by Maitra et al. [29]. They found that the critical Weber number for the droplet
82 impalement was independent of the substrate temperature. However, Alizadeh et al. [29]
83 reported a strong temperature dependency for the impact dynamics of water droplets on
84 hydrophilic to superhydrophobic surfaces at a Weber number of 138. Lower substrate
85 temperatures lead to less droplet retraction. Zheyang et al. [30] reported the detailed dynamic
86 motions of a water droplet impacting an ice surface and concluded that the lowering of the
87 ice surface temperature decreases the maximum spreading factor.

88 Much effort has been placed on investigating the impact behavior of water droplets on cold
89 superhydrophobic surfaces; however, most of these studies have been confined to narrow
90 parameter ranges. Mishchenko et al. [31] focused on the design of ice-free nanostructured
91 surfaces and evaluated droplet behavior on supercooled nano- and microstructured surfaces
92 able to repel impacting water before ice nucleation. They performed impact tests using 15
93 μL water droplets falling from 10 cm onto cold surfaces (-25 to -30 $^{\circ}\text{C}$) and found that
94 the rebounding process was suppressed on surfaces colder than -25 $^{\circ}\text{C}$. Ding et al. [32]
95 investigated the effect of superhydrophobic surface inclinations and the degree of
96 supercooling on water droplet dynamics. In their study, a 14 μL water droplet was projected
97 at 0.99 $\text{m}\cdot\text{s}^{-1}$ onto a superhydrophobic surface having a static CA of $160 \pm 1^{\circ}$. They
98 observed that the droplet successively underwent full rebound, partial rebound, and no

99 rebound as surface temperatures decreased. Zheng et al. [33] demonstrated that
100 supercooled droplets impacting inclined and dry superhydrophobic surfaces can bounce off
101 without freezing because of a reduced surface contact time and contact area of the
102 impinging water droplets on properly designed surfaces. Finally, Li et al. [34] investigated
103 the influence of a supercooled water droplet on cold hydrophilic and superhydrophobic
104 surfaces. They observed that solidification of a 1.6 mm diameter supercooled droplet
105 impacting a cold superhydrophobic substrate at $3.4 \text{ m}\cdot\text{s}^{-1}$ reduced droplet bouncing.

106 Most research in this area has focused on the outcome of droplet regimes relying on a few
107 select parameters. Critically, these studies lack an analysis incorporating all possible
108 parameters, especially as the droplet impact process is a complex interaction of multiple
109 variables, as discussed above. However, the development of machine learning-based
110 methods that can consider all influential parameters affecting impacting droplet behavior
111 can offer some design criteria for water-repellent superhydrophobic surfaces subjected to
112 various temperatures. Machine learning can support engineering tasks to manage and
113 extract insights from the resulting data [35,36]. Furthermore, these approaches can reduce
114 the high costs and time required for carrying out multiple complex experiments.

115 Artificial intelligence and statistical-learning methods are increasingly used in various
116 fields, such as computer science, material science, and aircraft icing research [37–39]. For
117 example, Zhang et al. [40] used artificial neural networks and evolutionary computation to
118 enhance our understanding of superhydrophobic surfaces by determining the relationship
119 between water droplet volume, nanoparticle weight, the falling distance between the
120 superhydrophobic surface and the water droplet, and multiple properties (droplet CA,

121 sliding angle, and adhesive force). Li et al. [41] also applied machine learning to predict
122 the severity of aircraft icing in relation to various conditions, including liquid water
123 content, droplet diameter, and exposure time. Although it is possible to examine the effect
124 of various parameters in conventional experimental investigations, machine-learning
125 models improve our ability to find patterns within large data sets.

126 There is currently no comprehensive study that has examined the influence of surface
127 characteristics on the dynamic behavior of water droplets at different temperatures while
128 incorporating machine learning approach. In our previous work [42], we investigated the
129 drop dynamic behavior on hydrophobic and superhydrophobic surfaces at room
130 temperature in line with finding some design criteria (in terms of surface CA, CAH, and
131 roughness values) based on machine learning approach to improve the feasibility of
132 achieving the bouncing of drops when they impact on hydrophobic and superhydrophobic
133 surfaces. The goal of the presented research is to investigate temperature dependency of
134 water droplet impact on hydrophobic and superhydrophobic surfaces and coupling the
135 experimental results with machine learning-based methods for prediction of the drop
136 dynamic behavior at different temperatures. It is worth mentioning that this new article
137 complements the previous article by examining the temperature dependency of the droplet
138 dynamic, using linear and nonlinear methods of machine learning to predict the droplet
139 dynamic, providing different equations for prediction the droplet behavior and studying the
140 relative importance of affecting parameters on water droplet impact.

141 In this paper, we study the temperature dependency of impact dynamics. We quantify the
142 influence of drop properties, kinematic parameters, and surface characteristics on

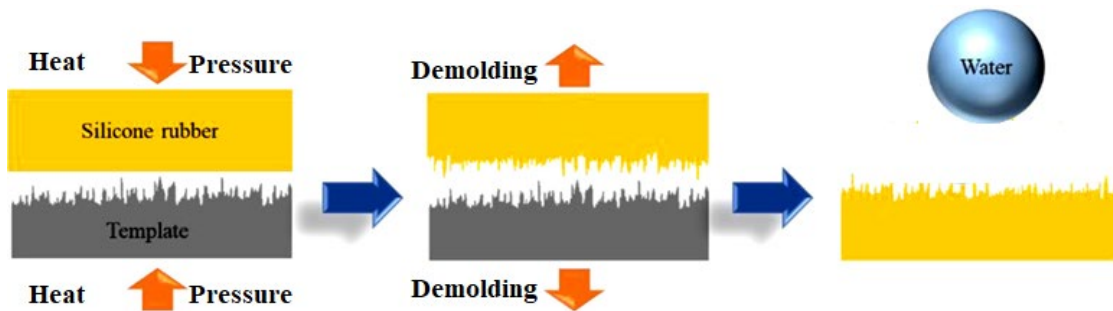
143 impacting droplet behavior (e.g., deposition, bouncing, and splashing) on silicone rubber
144 surfaces (hydrophobic to superhydrophobic). A machine-learning technique is applied to
145 evaluate the outcome regime of impact droplet behavior based on CA, CAH, temperature
146 (T), and the root mean square surface roughness value (S_q), We and Re numbers. We
147 demonstrate that machine learning can effectively predict droplet impact behavior. We
148 formulate three different methods using a decision tree, random forest, and logistic
149 regression to develop a data-driven approach for predicting droplet impact behavior by
150 exploring the complex interactions between CA, CAH, S_q , T, and the We and Re numbers.
151 Our experimentation and machine-learning approach is a novel means of investigating
152 droplet behavior on hydrophobic to superhydrophobic surfaces at different temperatures.
153 Moreover, we develop correlations through logistic regression for predicting the behavior
154 probability of impacting droplets as a function of the analyzed parameters. We selected
155 these three machine-learning methods for predicting impact droplet behavior, as they are
156 state-of-the-art techniques having a strong predictive capability.

157 **2. Material and Methods**

158 *2.1. Sample preparation*

159 High-temperature vulcanized (HTV) silicone rubber was used as the process material. A
160 chemical-etching method was used to produce microstructured aluminum templates
161 (A6061) using a 4.8, 9.8, 14.8, 19.8 wt% hydrochloric acid solution and immersion of
162 aluminum templates in this solution for 2 h. A micro-compression molding machine with
163 two temperature-adjustable platens (Carver Inc. USA) was used to mold the rubber
164 samples. The hydraulic press system can precisely control an applied force of 3 to 194 kN
165 [24]. The rubber material is cast in three pieces of flat molds, each with a right rectangular

166 prism cavity of $25 \times 15 \times 6 \text{ mm}^3$. After the template was placed in the cavity, the rubber
167 material was placed over it. Next, the top of the mold was closed. The mold was set in the
168 press machine to begin the process. In the press machine, the following mold parameters
169 were set: a molding pressure of 35 MPa, a curing time of 4.7 minutes, and a mold
170 temperature of $149 \text{ }^\circ\text{C}$. After the process was complete, the mold was opened, and the
171 silicone rubber with the size of $25 \times 15 \times 6 \text{ mm}^3$ was removed from the aluminum template
172 (Fig. 1).



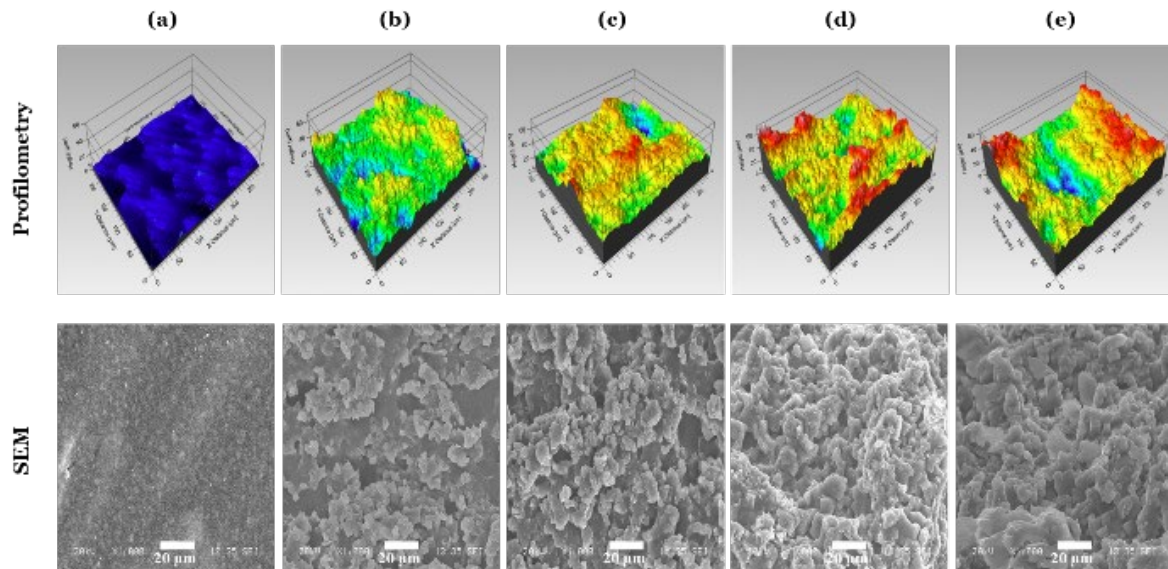
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174 Fig. 1 Schematic of the fabrication of micro-nanostructured silicone rubber surfaces via a
175 microcompression molding technique [21]

176 2.2. Surface Characterization

177 Using a Kruss™ DSA100 goniometer at $25 \text{ }^\circ\text{C}$, we measured the water CA using a water
178 droplet of $4\mu\text{L}$ deionized water based on the method of Young-Laplace. A CAH is equal
179 to the difference between the advancing and receding contact angles when a droplet moves
180 on the surface. All wettability measurements were taken at five different points on each
181 sample to ensure accuracy and reproducibility. For each sample the average and standard
182 deviation were reported. The surfaces were characterized using a confocal laser
183 microscopy profiler (Profil3D, Filmetrics, USA) and a scanning electron microscope

184 (JSM-6480 LV, JEOL Japan). In Fig. 2, the surface structures of different samples are
185 presented as SEM images and 3D profiles.

186 In the case of Sample1, which was replicated on a smooth aluminum template, it exhibited
187 hydrophobic surface characteristics (due to its low surface energy), while the other samples
188 exhibited superhydrophilicity (because they were replicated on aluminum templates with
189 varying acid concentrations). Therefore, to fabricate superhydrophobic surfaces, we used
190 a low-surface-energy material with intrinsic hydrophobicity, combined with a sufficient
191 level of surface roughness. In the Sample 2, 3, 4, and 5 samples, S_q increased by 1.87, 3.77,
192 4.49, and 4.28 respectively compared to sample1. Table 1 shows CA, CAH, and S_q of these
193 five microstructured surfaces.



194
195 Fig. 2 The 3D surface profiles and SEM images of samples (a) 1, (b) 2, (c) 3, (d) 4, and (e) 5 [42].

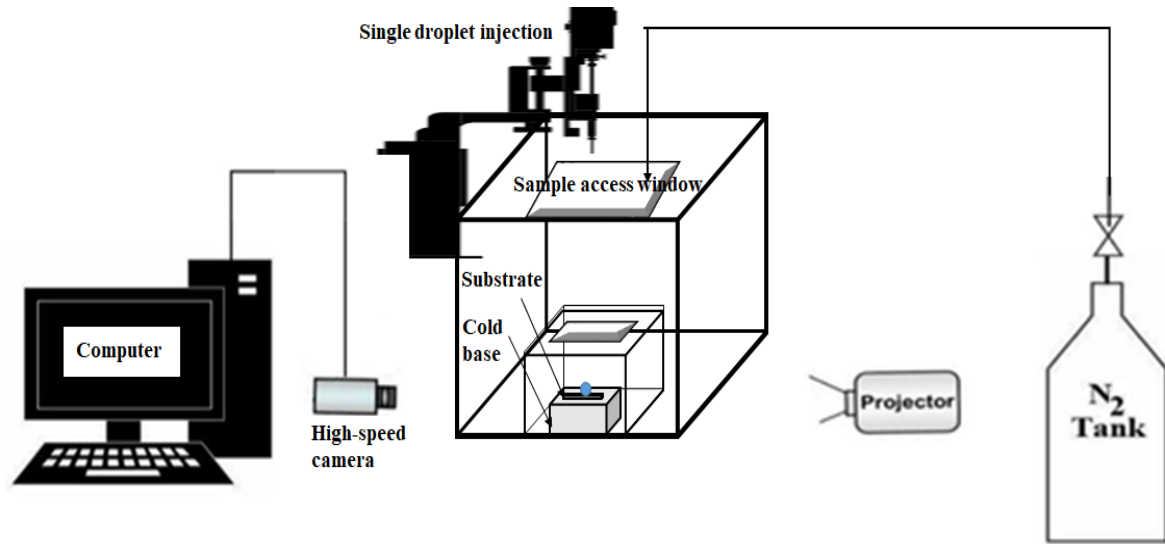
196 **Table 1** Surface characteristics of hydrophobic and superhydrophobic surfaces and
197 kinematic parameters used in the experiments. For all samples, falling velocity (u_0) varied
198 between 0.4 and 2.7 m/s, droplet size (d_o) was either 2.67 or 3.02 mm, and the surface
199 temperatures (T) were -20 , -10 , and 25 °C

Sample No.	CA (°)	CAH (°)	S _q (μm)
1	116.0 ± 2	46.5 ± 2.4	1.76 ± 0.17
2	154.5 ± 1.4	28.0 ± 1.6	3.29 ± 0.4
3	165.3 ± 1.1	1.5 ± 0.2	6.64 ± 0.32
4	166.6 ± 0.9	0.6 ± 0.3	7.90 ± 0.24
5	162.8 ± 0.9	1.3 ± 0.8	7.54 ± 0.33

200

201 2.3. Experimental Setup

202 The experimental freezing setup included a thermally insulating and optically transparent
203 chamber, high-speed camera, thermostatic bath, cold base, droplet injection system, test
204 samples, data acquisition system, temperature sensor, humidity sensor, and a vibration-free
205 table (Fig. 3). The double layer chamber that is thermally insulating and optically
206 transparent will be used to control the temperature of humidity of experiment and affecting
207 parameters on ice nucleation to be uniform during the experiment and increase the accuracy
208 and reproducibility of the experiments. Its transparency would facilitate imaging of the
209 freezing droplet. By having a small interior chamber (150 mm length × 150 mm width × 110
210 mm height) inside a large chamber (420 mm length × 420 mm width × 400 mm height), the
211 droplet could be subjected to a uniform environment. During the cooling process, the
212 relative humidity of the chamber can be controlled with a constant flow of dry nitrogen gas
213 (N₂). We adjusted the temperature of the cold base using a thermostatic bath. The chamber's
214 temperature and relative humidity were around 25 °C and 40% ± 3%, respectively. A high-
215 speed camera and LED illuminator monitored the droplet impact process and visually
216 recorded the droplet movement on the surfaces.



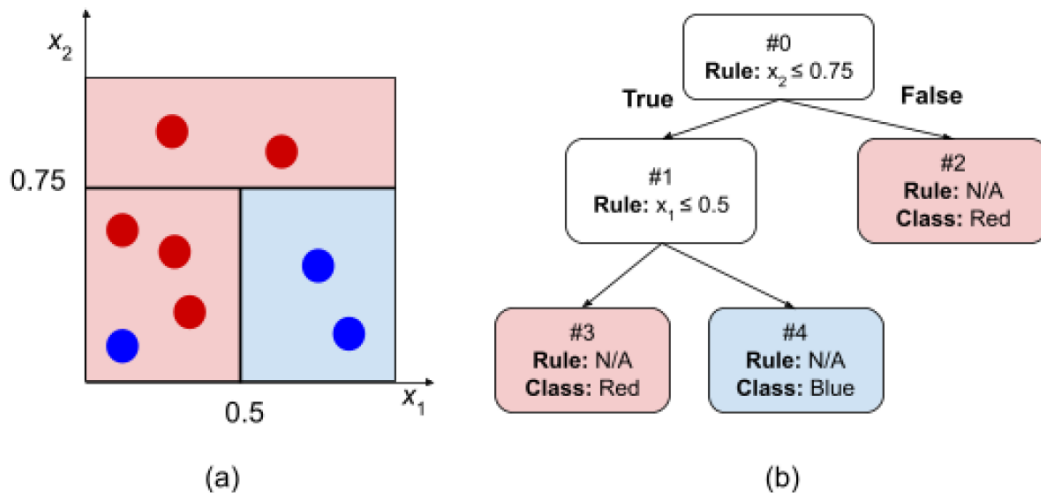
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218 **Fig. 3** Schematic of the experimental setup

219 We used two different syringe nozzles to vary drop diameters, all the spherical water
 220 droplets had a fixed volume of 10 μL (2.67 mm in diameter) and 20 μL (3.02 mm in
 221 diameter). We released 2.67 and 3.02 mm diameter droplets at room temperature from a
 222 droplet injection system at various heights, corresponding to initial droplet impact
 223 velocities ranging between 0.4 and 2.74 $\text{m}\cdot\text{s}^{-1}$. We placed a high-speed camera
 224 (MotionBLITZ, MIKROTRON, EoSens Cube 7, Germany) to record the droplet impact
 225 behavior at 3000 fps. Before each test, we adjusted the plate's temperature to either 25,
 226 -10 , or -20 $^{\circ}\text{C}$ and placed the sample on the plate. We repeated each test (a specific
 227 combination of parameters) at least three times. We computed the impact velocity of the
 228 droplets from the elapsed distance and time recorded by the camera and the ImageJ
 229 software [42]. All experimental parameters, including the droplet properties, surface
 230 characteristics, surface temperature, and impact velocity, are summarized in Table 1.

231 *2.4. Machine-Learning Methods*

232 Among the various machine-learning methods, regression and classification are the most
 233 commonly used methods. To predict impacting droplet behavior on the silicone rubber
 234 surfaces, we applied three machine-learning models: logistic regression (LR), decision tree
 235 (DT), and random forest (RF). DT machine-learning models partition the feature space into
 236 several decision regions by successively dividing the space through simple decision rules,
 237 where each rule can be as simple as thresholding a single feature [43]. Each decision region
 238 is then assigned a single class label based on the class of its training samples. When the
 239 tree is fully constructed, the class of any given test sample is predicted by identifying the
 240 decision region into which the sample falls. These models are applicable to multiclass and
 241 nonlinear classification problems.



242
 243 **Fig. 4.** An example illustrating DT classifier. (a) A 2D feature space corresponding to a data set
 244 with 2D feature vectors. The training samples are shown with red and blue circles with their color
 245 representing the class. The space is partitioned by a DT classifier trained over the training examples.
 246 (b) DT classifier that is trained over the training samples.

247 Fig. 4 shows an example of how a DT partitions 2-dimensional feature space of a 2D data
 248 with red (five labeled samples) and blue classes (three labeled samples). The left figure

249 (Fig. 4(a)) indicates the partitioning of the space, where the color of each region represents
250 the class label assigned to it (red shadow for class red and blue shadow for class blue). The
251 right figure (Fig.4(b)) shows the corresponding DT as a tree structure where each node
252 contains a rule in the form of a feature thresholding. If the rule is satisfied for any test
253 sample (True), we choose the left node in our next step, otherwise (False) we go to the right
254 node. Starting from the root node (#0), we keep choosing nodes based on the nodes' rules
255 until we get to a node without any children. Such nodes are usually called "leaves". Each
256 leaf is associated with a partition in the feature space and is assigned a class label based on
257 the class majority of the training samples belonging to that partition. In our example in Fig.
258 4, node #2 is a leaf that corresponds to the horizontal rectangle at the top of Fig. 4(a) which
259 has two red training samples, hence is assigned the red class. Any test that falls in this
260 partition (i.e., $x_2 \geq 0.75$) will be predicted as a "red" sample.

261 Training a decision tree starts from the root node corresponding to the entire feature space,
262 where a feature and a cut-off threshold are selected so that the selected feature partitions
263 the data into two groups having the highest possible sample homogeneity. Each partition
264 forms a new node in the tree, where the same procedure is repeated to further divide the
265 data set into two additional leaves. The divisions continue until either all the paths have
266 reached the maximum number of divisions from the root, i.e., maximum depth (denoted by
267 d_{\max}), or all the leaves are assigned partitions with a sample homogeneity higher than a pre-
268 specified threshold. Nodes at which splitting is stopped are referred to as leaves. When
269 classifying the groups of data, the homogeneity of samples is inversely defined on the basis
270 of the impurity of their class labels. Lower impurity implies higher homogeneity, and the

271 smallest impurity is achieved when the class labels of all samples are identical. In our
272 experiments, we used the Ginni index to measure class impurity, defined as:

$$273 \quad Ginni = 1 - \sum_{i=1}^c P_i^2, \quad \text{Eq. 1}$$

274 where P_i denotes the empirical distribution of the i^{th} class. We also ensured that all leaves
275 contained at least one sample.

276 Training the DT can easily lead to an over-partitioning of the feature space, which, in turn,
277 implies overfitting and instability. A common approach to alleviate this issue is to build an
278 ensemble of trees via bootstrapping, a method commonly referred to as random forest (RF).
279 This method consists of constructing multiple DTs, each separately trained with a distinct
280 randomized data set that is obtained by sampling with replacement from the original data
281 set [44]. Each tree in the forest will make an individual prediction of the class label of a
282 test sample, and the final prediction will be reported through majority voting. In contrast
283 to individual trees, RFs can assign a probability to their inferences, enabling an uncertainty
284 analysis of the results. In our experiments, we trained 50 DTs to build our RF.

285 For an alternative approach, we also tried logistic regression (LR) in our experiments as a
286 linear classifier that is widely used because of its simplicity [45]. This model is a single-
287 layer neural network where the output undergoes a softmax function (a function in form of
288 $e^{z_i} / \sum_{j=1}^n e^{z_j}$ for normalizing a set of real-valued scores z_1, \dots, z_n) to generate a probability
289 distribution, which is to be interpreted as the probability of belonging to different classes.
290 Compactly representing this model, given the input vector $x = [x_1, \dots, x_6] = [T, Re, We, CA,$
291 $CAH, S_q]$, LR produces the output as the probability that x belongs to class i ($1 \leq i \leq 6$):

292 $P(y = i | x) = e^{\beta_{i1}x_1 + \dots + \beta_{id}x_d} / \sum_{j=1}^6 e^{\beta_{j1}x_1 + \dots + \beta_{jd}x_d}$, Eq. 2

293 where $\beta_i = [\beta_{i0}, \beta_{i1}, \dots, \beta_{id}]$ denotes the parameters corresponding to the i^{th} class. The
 294 exponents of these class probabilities are also known as the log-likelihood of the data
 295 sample x given the parameter values β_i , $1 \leq i \leq 6$. In the training step, parameters are
 296 tuned by maximizing (through the gradient ascent) the summation of the log-likelihood
 297 terms of the training data given their observed class labels plus a regularization penalty
 298 term that is usually in the form of the L1 [46] or L2 [47] norm of the parameter vectors.
 299 We used the latter in our experiments with a regularization coefficient of 1. Note that log-
 300 likelihoods are linear with respect to the data features; hence, logistic regression is known
 301 to have a lower class separability power than DT and RF, which experience greater
 302 nonlinearity.

303 **2.4.1. Prediction Rules**

304 *2.4.1.1. Decision Tree*

305 Class prediction in DT models includes following a set of simple decision rules over
 306 individual features. To keep our discussion concise, here we visualize only our shallowest
 307 DT model ($d_{\max} = 4$) and demonstrate how class is inferred through its branches and leaves.
 308 The tree structure and trained parameters of this model are shown in Fig. 5, in which each
 309 tree node (*rectangle*) is labeled by its parameters and training data statistics. The first line
 310 of each node specifies the node ID. The second line shows the node’s decision rule as an
 311 inequality for an individual feature. These inequalities divide the feature space into two
 312 halves producing one “child” node per partition. Moreover, a set of multiple inequalities
 313 collectively defines a partition (subregion) of the feature space. Each node in our tree is

314 assigned a feature space partition that is determined by the set of its ancestors' inequalities.
315 For instance, the root is assigned the whole feature space because it has no ancestors
316 (hence, no partitioning), and the partition for Node #3 is defined by the inequalities $\{S_q \leq$
317 $3.281, We \leq 87.925\}$. The leaves do not have decision rules as they lack descendants. The
318 third line in each node indicates the Ginni index value of the class labels for all the training
319 samples falling inside the node's partition. Two more properties are present inside the
320 leaves (i.e., nodes without descendants): "class sizes," explicitly listing the class
321 distribution of the training samples inside the corresponding partitions, and "class,"
322 representing the predicted class inferred by the leaf. The inferred class is determined by the
323 class having the largest number of training samples in the leaf's partition. Furthermore,
324 class probabilities are obtained by estimating the empirical distribution from the class sizes.

325 Decision-tree predictions (class assignment) for any given test sample begin from the root
326 node (#0), shown at the top of the tree, and move forward sequentially by choosing the
327 next node based on the current node's decision rule. This sequential decision-making task
328 ends once it reaches a leaf. Using Fig. 5, we can follow, as an example, the test sample
329 with the feature values $x = [T, Re, We, CA, CAH, S_q] = [-10, 3303.94, 52.22, 166.6, 0.6,$
330 $7.9]$. Starting from the root node, the first decision rule we consider is $S_q \leq 3.281$. Given
331 that our test's S_q is 7.9, the rule's inequality does not hold, and therefore, we choose Node
332 #2 (the child node on the right in the figure) as our next node. The decision rule for Node
333 #2 is $We \leq 87.925$, which does hold in the case of our test sample ($We = 52.22$). Therefore,
334 we choose Node #3 (the left child of Node #2 in Fig. 5) as our third step. Repeating this
335 procedure, we encounter the decision rule $T \leq 24.843$ in Node #3, which holds; thus, we
336 move to Node #4, followed by decision rule $Re \leq 3526.178$, which also holds; hence we

337 move to Node #5. At Node #5, our prediction procedure ends because there are no further
338 descendants. The predicted class in this leaf is PB (Fig. 5). To measure the uncertainty of
339 this prediction, we can compute the empirical class distributions by normalizing the class
340 sizes such that they sum to one. From Fig. 5, the class sizes are [0, 55, 0, 297, 8, 0] (the
341 total number of training samples in Node #5's partition is $55 + 297 + 8 = 360$).
342 Normalizing, this yields the empirical distribution $[0, 55/360, 0, 297/360, 8/360, 0] = [0,$
343 $0.153, 0, 0.825, 0.022, 0]$ corresponding to classes BS, D, FB, PB, PS, and S, respectively.
344 Hence, our final prediction for this test sample will be class PB with a probability of 0.825.

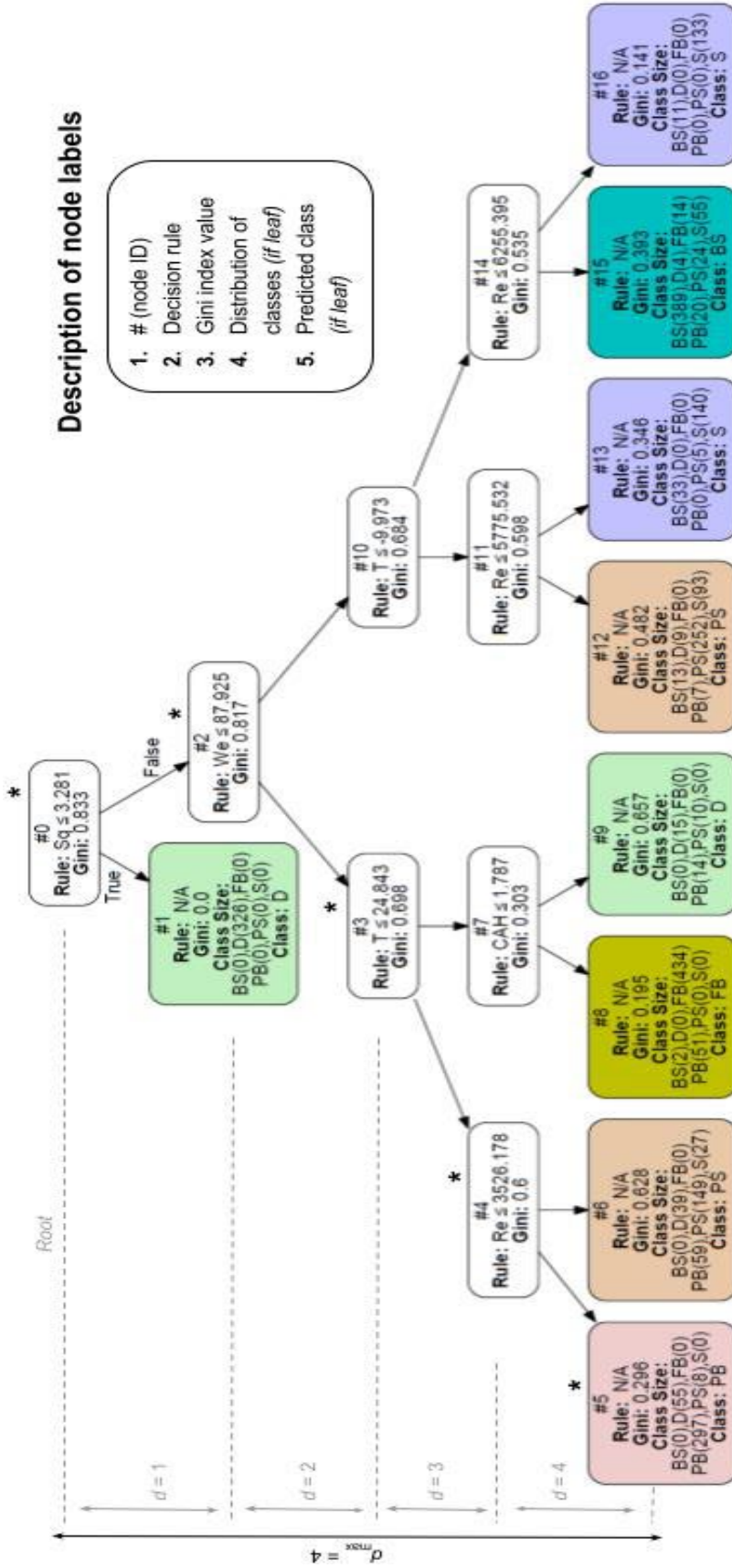


Fig.5 Tree structure and parameter values of our trained decision tree (DT) ($d_{max} = 4$). The asterisks (over selected nodes) indicate the decision path of the class inference of the exemplary test sample discussed in Section 2.4.1.1

346 2.4.1.2. *Logistic Regression*

347 The class probabilities in this model are described through softmax functions. From the
348 trained parameters and the equation above (Eq. 2), the class probabilities can be described
349 as:

$$P(y=D | x) \propto \exp(0.004 - 0.097 T - 0.003 Re + 0.064 We + 0.004 CA + 0.454 CAH - 0.021 S_q), \quad \text{Eq.3}$$

$$P(y=PB | x) \propto \exp(0.317 T - 0.002 Re - 0.049 We + 0.044 CA - 0.23 CAH + 0.044 S_q), \quad \text{Eq.4}$$

$$P(y=FB | x) \propto \exp(-0.002 - 0.079 T + 0.005 Re - 0.046 We - 0.103 CA - 0.09 CAH - 0.024 S_q), \quad \text{Eq.5}$$

$$P(y=PS | x) \propto \exp(-0.023 T - 0.003 Re + 0.04 We + 0.089 CA - 0.066 CAH), \quad \text{Eq.6}$$

$$P(y=BS | x) \propto \exp(-0.119 T - 0.001 Re + 0.032 We + 0.029 CA + 0.073 CAH - 0.022 S_q), \quad \text{and} \quad \text{Eq.7}$$

$$P(y=S | x) \propto \exp(-0.001 + 0.002 T + 0.004 Re - 0.041 We - 0.064 CA - 0.141 CAH + 0.218 S_q). \quad \text{Eq.8}$$

350 Note that these equations do not indicate equalities. The right-hand side of each equation
351 is proportional to the corresponding class probability. The exact class probabilities can be
352 obtained by normalizing these values by their summation. To predict the class label of a
353 test sample, we can simply compute all the right-hand sides of the equations and then select
354 the class having the highest score among all the classes.

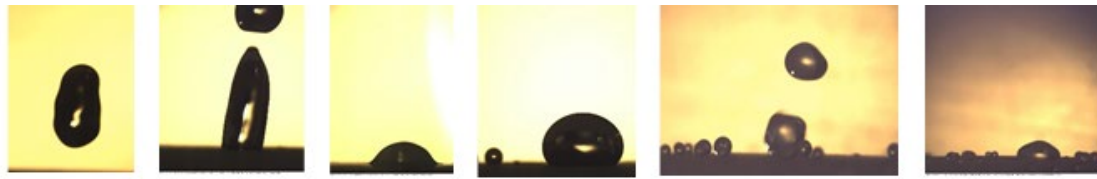
355 These LR-based equations are a powerful means of modeling multilabel outcomes, such as
356 the various phenomena (deposition, full bouncing, partial bouncing, and splashing, etc.)
357 that occur during a droplet impact on a solid surface to measure the statistical significance
358 of each independent variable with respect to probability. To better understand how to make
359 class predictions with LR, let us consider a test sample with the feature $x = [T, Re, We, CA,$

360 CAH, S_q] = [25, 4237.42, 85.9, 162.8, 1.3, 7.54]. Inserting these values into the class
361 probability equations produces the non-normalized class scores: [91.65, 0.00, 11.44, 94.84,
362 0.00, 0.16], implying that the fourth class (i.e., PS) is assigned the largest score and is
363 therefore the predicted class. The second and fifth classes (D and BS, respectively)
364 obtained near-zero scores. Normalizing these values by division to their summation will
365 give us a probability distribution that can be interpreted as the certainty of our trained LR
366 model regarding the predicted class. For this test example, the probability distribution is
367 [0.46, 0.00, 0.06, 0.48, 0.00, 0.00], implying that our model probabilistically infers that this
368 sample belongs to Class 1 (S) with a probability of 46%, Class 3 (PB) with a probability
369 of 48%, and Class 4 (PS) with a probability of only 6%.

370 **3. Results and Discussion**

371 *3.1. Outcome of Droplet Impact Dynamics*

372 We first investigated the influence of surface temperature on impact dynamics by varying
373 the substrate temperature (-20, -10, and 25 °C) while maintaining the initial droplet
374 temperature at room temperature. For the 25 °C experimental results, we relied on some
375 previously published data from our lab [42]. As illustrated in Fig. 6, the water droplets vary
376 in their impact behaviors on the surfaces, including full bouncing (FB), partial bouncing
377 (PB), deposition (D), prompt splashing (PS), bouncing-splashing (BS), and splashing (S)
378 [23].



379 **FB** **PB** **D** **PS** **BS** **S**

380 **Fig. 6** Droplet impact regimes on silicone rubber surfaces. Regimes include full bouncing
 381 (FB, $T = 25\text{ }^{\circ}\text{C}$, $S_q = 7.54\text{ }\mu\text{m}$, $We = 15.81$), partial bouncing (PB, $T = -10\text{ }^{\circ}\text{C}$, $S_q = 6.64$
 382 μm , $We = 45.25$), deposition (D, $T = -10\text{ }^{\circ}\text{C}$, $S_q = 1.76\text{ }\mu\text{m}$, $We = 16.81$), prompt splashing
 383 (PS, $T = -10\text{ }^{\circ}\text{C}$, $S_q = 7.9\text{ }\mu\text{m}$, $We = 62.37$), bouncing-splashing (BS, $T = -10\text{ }^{\circ}\text{C}$, $S_q = 7.9$
 384 μm , $We = 142.62$), and splashing (S, $T = -10\text{ }^{\circ}\text{C}$, $S_q = 7.54\text{ }\mu\text{m}$, $We = 208.74$)

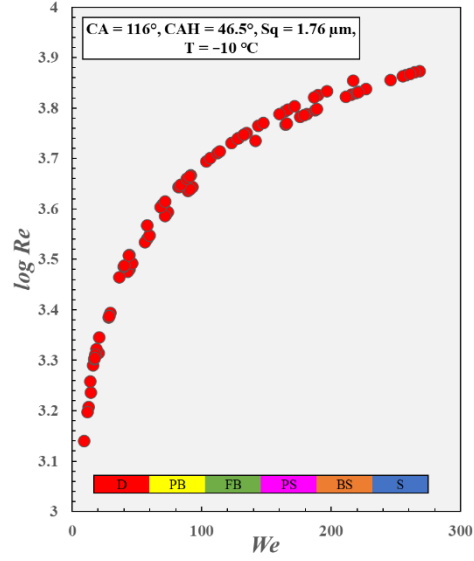
385 Impacting a droplet on hydrophilic, hydrophobic, and superhydrophobic surfaces alters the
 386 resulting regime in terms of deposition, bouncing, splashing, etc. Spreading or sticking of
 387 droplets can be observed when the test surfaces are hydrophilic or hydrophobic. In contrast,
 388 droplet mobility increases on superhydrophobic surfaces having a low CAH. The kinetic
 389 energy of the impacting droplet on the hydrophobic and superhydrophobic surfaces is
 390 converted into surface energy, and a small amount of energy is lost through viscous
 391 dissipation energy. Balances between inertia, viscosity, and capillary forces control the
 392 dynamic of droplets [20]. Surface properties, such as wettability, roughness, and
 393 temperature, markedly affect the bouncing and deposition of droplets; at lower
 394 temperatures, for example, the losses from viscous dissipation increase and lead to less
 395 available energy for bouncing. Therefore, the probability of bouncing decreases at lower
 396 temperatures [48].

397 Impacting water droplets at cold temperatures (-10 and $-20\text{ }^{\circ}\text{C}$) operated in five regimes,
 398 i.e., deposition, partial bouncing, splashing, prompt splashing, and a transition regime
 399 between bouncing and splashing (Fig. 7). Fig. 7a1, a2 shows the dimensionless parameters
 400 for droplets hitting a hydrophobic surface ($CA = 116^{\circ}$, $CAH = 46.5^{\circ}$). For all Re and We

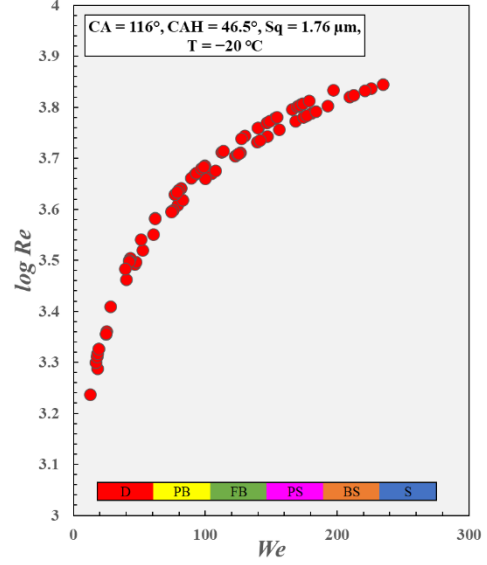
401 numbers between 1380 and 7480 and 9 and 267, respectively, deposition occurs, and the
402 liquid droplet cannot rebound. At lower Re and We numbers ($Re < 5293$ and $We < 134$ at –
403 10 °C, $Re < 4646$ and $We < 103$ at –20 °C) for a non-water-repelling superhydrophobic
404 surface of roughness 3.29 μm , the deposition of droplets and a prompt splashing occurs
405 (Fig. 7b1,b2)), whereas at higher Re and We numbers, the impinging droplets splash.

406 Droplet impingement on water-repellent superhydrophobic surfaces having a lower CAH
407 and varying roughness values (6.64, 7.54, and 7.9 μm) is shown in Fig. 7c–e, respectively.
408 Depending on the surface roughness value, partial bouncing, deposition, splashing, prompt
409 splashing, and bouncing-splashing can occur. At lower Re and We numbers, the droplets
410 experience partial bouncing on these water-repellent surfaces. At intermediate Re and We
411 numbers, prompt splashing occurs, and eventually, the droplets show a splashing behavior
412 at higher Re and We numbers. Droplets were not deposited during the impact process for
413 the superhydrophobic surface with the highest CA, highest roughness, and lowest CAH,
414 although the probability of bouncing-splashing increases as surface roughness is greater
415 for these surfaces at –10 °C.

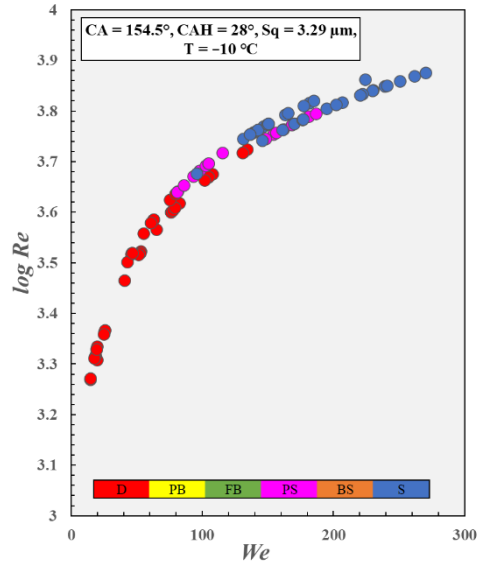
(a1)



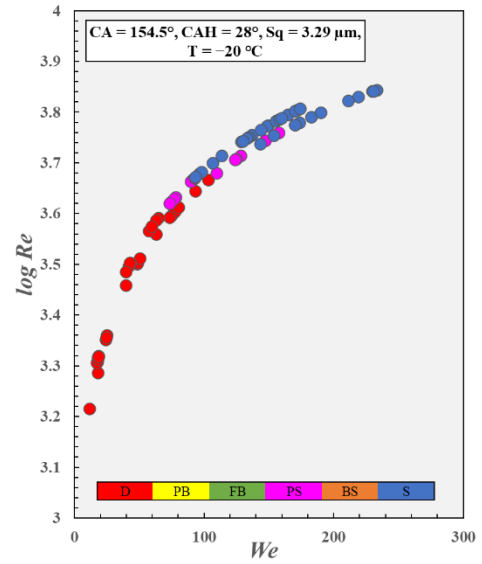
(a2)



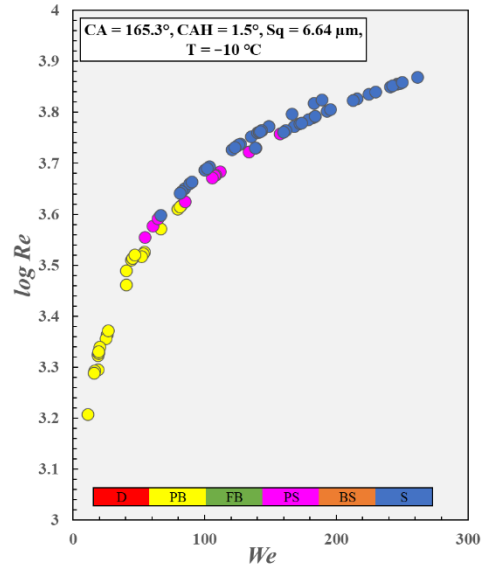
(b1)



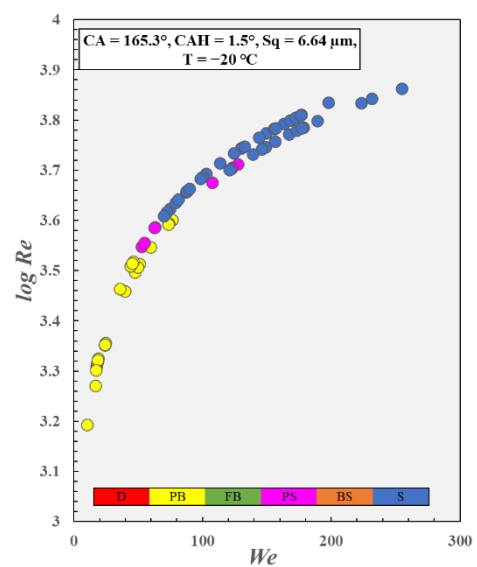
(b2)

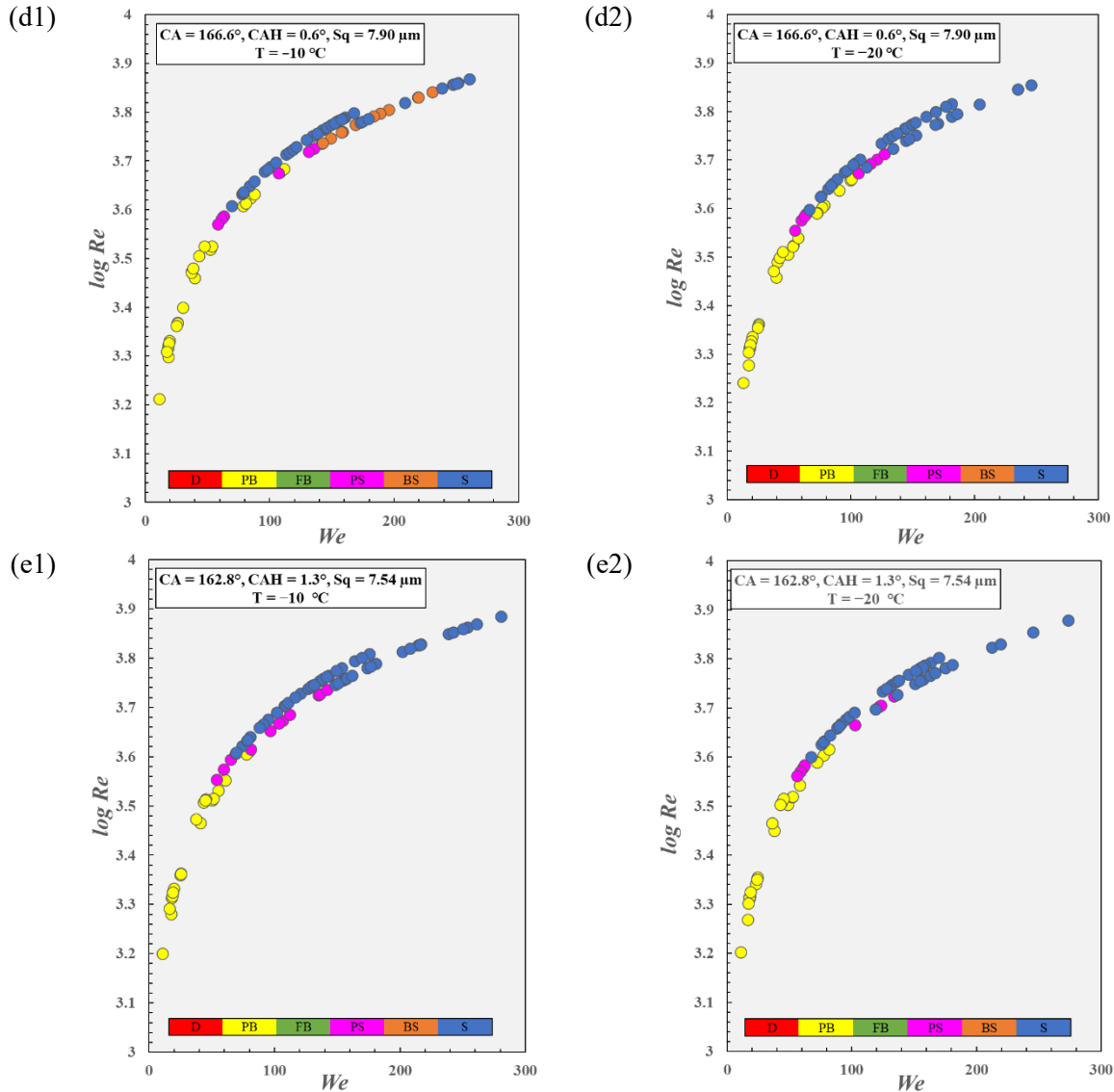


(c1)



(c2)





416 **Fig. 7** Impact dynamics on silicone rubber surfaces: **a1** S_q = 1.76 μm , T = -10 °C; **a2** S_q = 1.76 μm ,
 417 T = -20 °C; **b1** S_q = 3.29 μm , T = -10 °C; **b2** S_q = 3.29 μm , T = -20 °C; **c1** S_q = 6.64 μm , T = -10
 418 °C; **c2** S_q = 6.64 μm , T = -20 °C; **d1** S_q = 7.90 μm , T = -10 °C; **d2** S_q = 7.90 μm , T = -20 °C; **e1**
 419 S_q = 7.54 μm , T = -10 °C; **e2** S_q = 7.54 μm , T = -20 °C

420 Fig. 8 a–e shows the impacting droplet behaviors at three surface temperatures for five
 421 different hydrophobic and superhydrophobic substrates. We observe that the effect of
 422 temperature is negligible for hydrophobic substrates and for non-water-repelling
 423 superhydrophobic surfaces having a roughness of 3.29 μm (Fig. 8 a, b). Impacting droplets
 424 are not affected by low temperatures on substrates 1 and 2. For the droplet impact on
 425 hydrophobic substrate 1, all operating conditions promote the spreading of the droplet;

426 however, for non-water-repelling superhydrophobic substrate 2, having a lower Weber
427 number (<130), deposition occurs.

428 When observing impacting droplets on water-repellent superhydrophobic substrates (Fig.
429 8 c–e), we note that at low droplet velocity, the superhydrophobic surfaces of varying
430 roughness (6.64, 7.54, and 7.9 μm) had better water repellency at sub-zero temperatures.
431 These water-repellent superhydrophobic substrates reduce the probability of rebounding at
432 lower temperatures. Droplets can partially or completely bounce off all water-repellent
433 superhydrophobic surfaces at temperatures above 0 $^{\circ}\text{C}$ for $We <110$ and $Re <5000$;
434 however, as the temperature of the surfaces is reduced, droplets show partial bouncing on
435 the same substrates (having the same We values of <110), which related to the viscous
436 dissipation.

437 Increasing the impact velocity of the droplet on rough surfaces heightens the probability of
438 splashing (Fig. 8), as has been reported in other studies [49,50]. However, increased droplet
439 velocity reduces the probability of bouncing, particularly at low temperatures; for example,
440 we did not find conditions in which full bouncing could be observed at low temperatures.
441 In contrast, at room temperature, we observe the complete retraction of water on water-
442 repellent superhydrophobic surfaces (green symbol in Fig. 8). The most commonly
443 observed behaviors of droplet dynamics on superhydrophobic surfaces at room temperature
444 are complete bouncing, partial bouncing, and splashing [21].

445 Temperature, therefore, has a minor effect on droplet dynamics on cold hydrophobic and
446 superhydrophobic silicone rubber surfaces for a wide range of We numbers and substrate
447 temperatures. A possible reason for this temperature-independent nature of droplet

448 impacting is the delay of solidification. A spreading velocity greater than that for
449 solidification, even in the case of solidification at the early stages of drop impact, leads to
450 this limited influence of sub-zero temperatures on droplet impact [34].

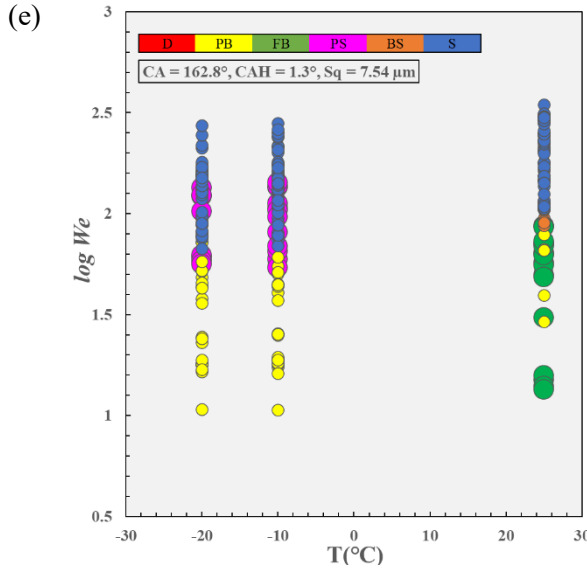
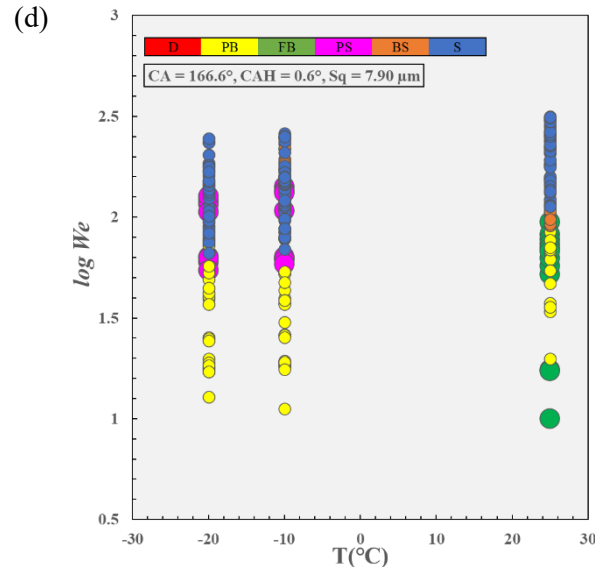
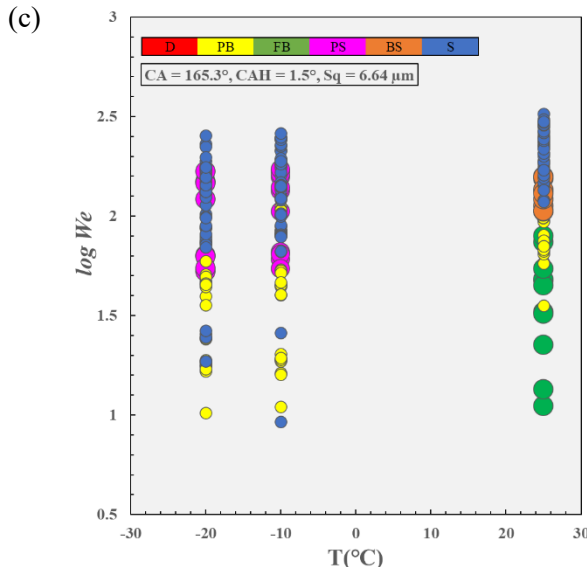
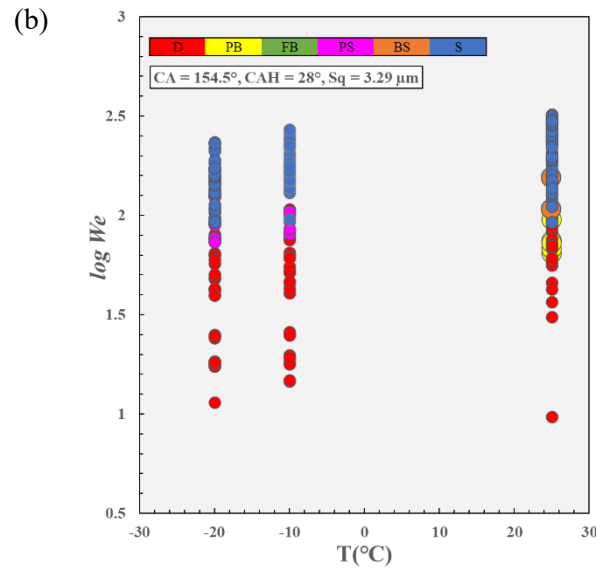
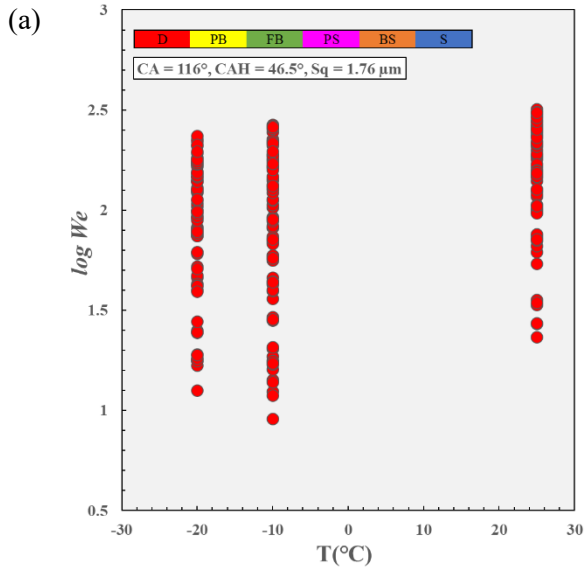
451 Another reason relates to heat transfer [48]. A rougher superhydrophobic surface
452 characterized by a low CAH can entrap more air pockets in the interface between the
453 surface and the water droplet. This entrapment reduces the contact area with the droplet
454 and acts as an insulator to heighten the heat transfer barrier. The reduced contact area and
455 heat transfer ability of superhydrophobic surfaces efficaciously limit increases in viscosity
456 by decreasing the temperature. The reduced heat transfer from a cold hydrophobic or
457 superhydrophobic surface to a droplet at high We numbers can limit water droplet dynamics
458 on these surfaces. Therefore, substrate temperature has a similar but weaker effect on the
459 superhydrophobic substrates at higher droplet velocities.

460 Moreover, the effect of surface temperature on droplet bouncing at lower We numbers for
461 water-repellent superhydrophobic silicone rubber surfaces can be explained by the extent
462 of energy dissipation and the wetting transition within the surface structures. Lower
463 temperatures increase the viscosity of droplets, thereby increasing contact time and viscous
464 dissipation while reducing the probability of bouncing. Moreover, surface wettability is
465 significantly influenced by temperature. The wetting transition of a Cassie-Baxter to a
466 Wenzel state by decreasing the temperature can reduce droplet bouncing [21,51].

467 Therefore, unlike many previous studies that examined the impact behavior of water
468 droplets on cold superhydrophobic surfaces with a limited set of parameters, we
469 investigated the effects of a wider range of influencing factors [31–34]. In general, for a

470 wide range of We and Re numbers, our experimental results showed that the temperature
471 has a minor impact on droplet dynamics on cold hydrophobic or superhydrophobic silicone
472 rubber surfaces while previous studies reported a strong temperature dependency for the
473 impact dynamics of water droplets [30,31,48]. Increasing We number and surface
474 roughness, heightens the probability of splashing, as has been reported in other studies
475 [49,50,52].

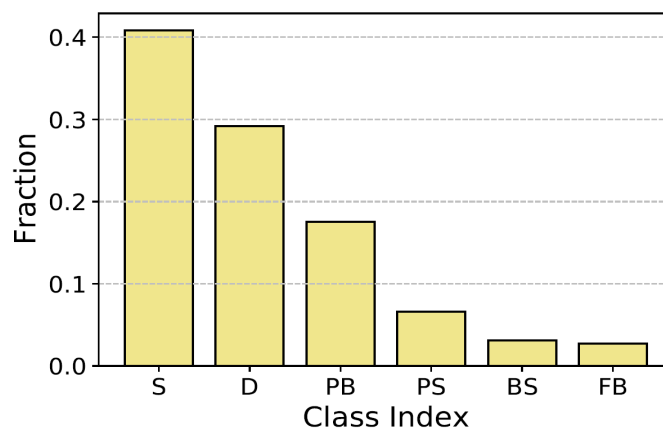
476 We then examine all possible variables simultaneously using machine-learning methods in
477 the next section to analyze droplet regime; however, many of works do not include an
478 analysis that incorporates multiple factors simultaneously [48,53,54]. Many studies have
479 reported the use of machine learning techniques in material science, superhydrophobicity
480 and icephobicity [40,41,54,55]. In this work we describe to the best of our knowledge the
481 first application of machine learning to the detailed dynamic of water droplets impacting
482 hydrophobic and superhydrophobic surfaces at different temperatures. Although the
483 number of publications focused on droplet impact dynamics has increased recently, various
484 aspects need study for the design of high-performance technical devices. The complexities
485 of the impact process and the interaction of various influencing parameters could show the
486 promise of the machine-learning approach. Therefore, if the dynamic behavior of droplets
487 can be predicted before conducting experiments, assessing the performance of droplet-
488 based devices and industrial applications can be done more accurately and effectively.



489 **Fig. 8** Droplet impact dynamics regime map for silicone rubber surfaces **a** $S_q = 1.76$; **b** S_q
490 $= 3.29$; **c** $S_q = 6.64$; **d** $S_q = 7.90$; and **e** $S_q = 7.54 \mu\text{m}$ at different temperatures

491 3.2. Classification Results

492 We trained and evaluated the three classifiers on a data set that we obtained from our
493 experiments of impacting droplet behavior on silicone rubber surfaces (hydrophobic to
494 superhydrophobic surfaces). Our data set consisted of six-dimensional feature vectors (T ,
495 Re , We , CA , CAH , and S_q) and six classes (FB, PB, D, PS, BS, and S). We randomly split
496 the data into training and test partitions; the training set served for fitting the models, and
497 the test data were kept for evaluating the model. The impact drop regimes show a highly
498 imbalanced distribution (Fig. 9). To correct this imbalance between regimes, we
499 augmented the training data using the *synthetic minority oversampling technique* (SMOTE)
500 [56]. Oversampling ensures an equal number of training samples for each class; however,
501 the test data were untouched and hence remained with an uneven distribution between
502 classes. We had 2,688 training samples and 257 test samples following this preprocessing
503 step.



504 **Fig. 9** Histogram of classes within the initial data set before the use of SMOTE. Splashing
505 (S) is the dominant class (class 1), whereas bouncing-splashing (BS, class 2) and full
506 bouncing (FB, class 3) are the least represented classes. Other classes are partial bouncing
507 (PB, class 4), deposition (D, class 5), and prompt splashing (PS, class 6)
508

509 We used precision and recall to evaluate the performance of the trained classifiers. These
510 criteria are specifically designed to be used with imbalanced data sets. In a binary
511 positive/negative classification scenario, precision indicates the portion of the model's
512 positive predictions that truly belong to the positive class. Recall measures how well the
513 predictions cover the entire positive class. A low precision implies a high rate of false
514 positives, whereas a low recall indicates many false negatives within the predictions. To
515 summarize these two criteria, we used the F_1 score, defined as the harmonic mean of the
516 two. This score is also originally proposed for binary classification problems and is defined
517 as $F_1 = \frac{2 Pr.Rc}{(Pr+Rc)}$, where $Pr = \frac{TP}{TP+FP}$ and $Rc = \frac{TP}{TP+FN}$ are precision and recall, respectively,
518 and TP, FP, and FN denote the number of true positives (test samples with positive class
519 in reality and in prediction), false positives (test samples with negative class in reality but
520 positive in prediction), and false negatives (test samples with positive class in reality but
521 negative in prediction), respectively. For multiclass problems like ours, F_1 is reported as
522 the (weighted) average of F_1 scores individually evaluated for one-versus-rest of each class.
523 In the case of weighted averaging, the score of each class is weighted according to its
524 sample size, hence taking into account the existing class imbalance.

525 Table 2 summarizes the evaluation results of each classifier involved in our experiments.
526 As is shown by our results, all the machine learning models were capable of predicting
527 class labels though with different levels of accuracy; however, these results confirm the
528 inferior classification ability of the linear LR model, showing that this model could not
529 separate the training samples accurately and demonstrates the nonlinearity of the decision
530 boundaries between the classes in our data sets. The model yielding the highest test
531 accuracy had the highest complexity (non-linearity), which was expected as classifying

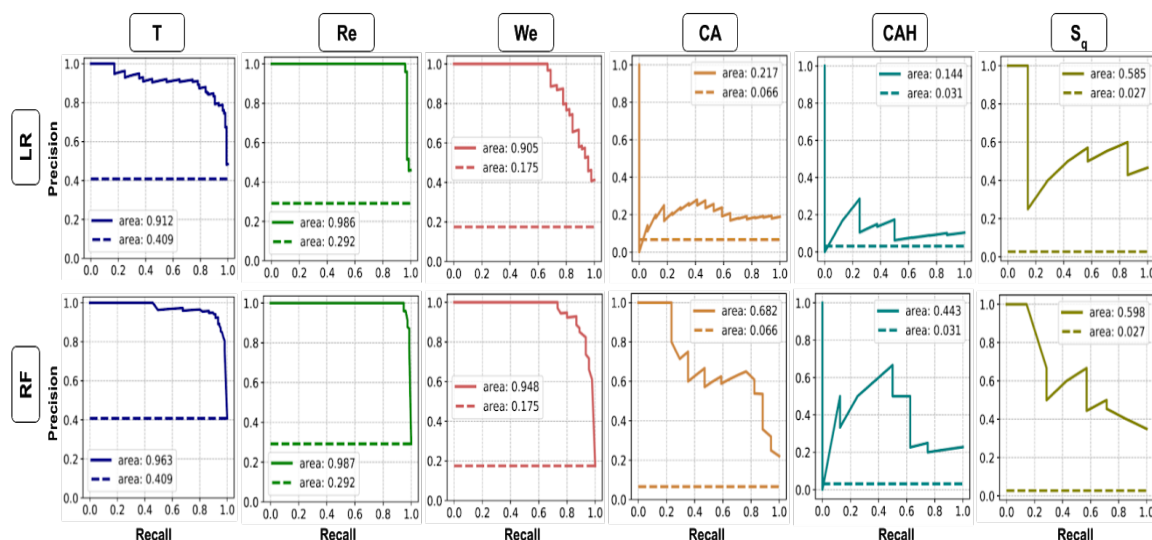
532 real-world data usually demands highly non-linear models. Now as the complexity of
 533 learning models grows, interpretability of their outcomes become more challenging. More
 534 specifically, if one needs to translate the resulting prediction into a handful of simple
 535 algebraic rules, one would have to apply simpler (or even linear) techniques. Such
 536 interpretability power comes with a cost, i.e., a more modest generalization accuracy. Some
 537 relatively shallow DT (e.g., $d_{\max} = 7$) are competitive against the deeper unconstrained DT
 538 ($d_{\max} = \text{NA}$) or RF models in classifying the unseen test samples; however, they severely
 539 underperform when explaining the training data set. This observation indicates that the
 540 generalizability of classification models does not always linearly increase with their
 541 complexity.

542 **Table 2** F_1 scores of the trained classifiers. Note that for the training data set, the weighted
 543 and unweighted average F_1 are the same because the class labels have been balanced
 544 through oversampling. Decision-tree (DT) models are trained using different d_{\max} values
 545 and without any depth-dependent conditions ($d_{\max} = \text{NA}$)

Metric (%)	DT (evaluated for various d_{\max})								RF	LR
	4	5	6	7	8	9	10	NA		
Training										
F_1 (av.)	79.3	82.89	88.44	91.3	93.73	95.38	97.0	100	100	77.3
Test										
F_1 (av.)	63.15	63.41	67.91	74.18	71.08	69.66	71.88	73.54	75.19	60.21
F_1 (weighted av.)	74.6	72.99	80.99	86.05	85.47	85.3	86.11	87.44	89.04	77

546 In calculating the F_1 score, the predictions are obtained by selecting the class having the
 547 highest probabilities computed through the predictive model. For instance, computing F_1
 548 in a binary classification is associated with placing a threshold for the positive class
 549 probabilities at 0.5. To obtain a more detailed portrait of the performance of a binary
 550 predictive model, we can change this threshold from 0 to 1 to obtain a range of F_1 scores
 551 or precision-recall pairs. Plotting the resulting precision versus the recalls yields what is
 552 usually called the precision-recall curve. An ideal classifier results in a recall and precision

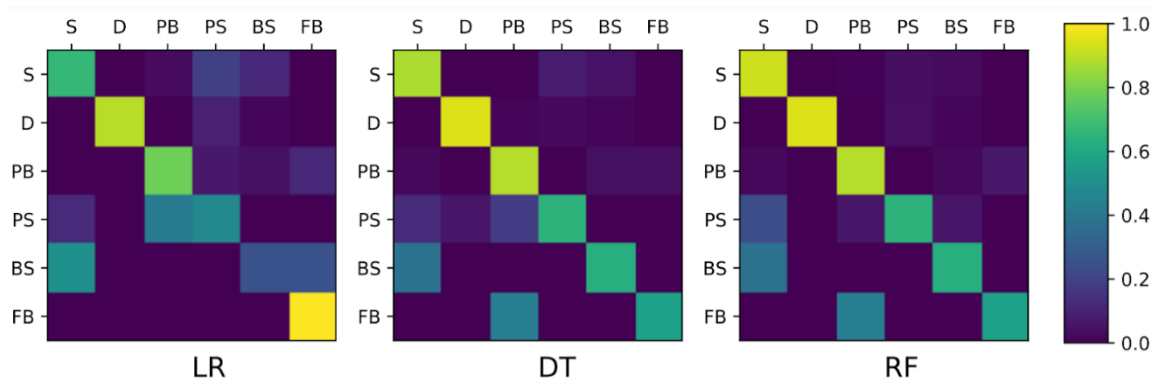
553 unit that is independent of the applied threshold; hence, the area under its precision-recall
 554 curve is 1. Fig. 10 presents the precision-recall curves of the LR and RF classifiers
 555 operating in a one-versus-rest mode for each class. DT is not considered here because trees
 556 that are not too shallow (e.g., $d_{\max} = \text{NA}$) typically assign degenerative class probabilities
 557 to samples (therefore, zero uncertainty); there are thus very few operating points on the
 558 precision-recall curve. We then compared our classifiers with random baselines, for which
 559 the precision remains constant and equal to the size ratio of the corresponding class
 560 (equivalent to the positive-to-negative class ratio of the corresponding one-versus-rest
 561 binary classification).



562 **Fig. 10** Precision-recall curves of the logistic-regression (LR) and random-forest (RF)
 563 classifiers. Each graph indicates the curves for binary classification of an individual class
 564 versus the rest (one-versus-rest). The dashed lines represent random classifiers that have
 565 constant precision-recall curves for any classification problem

566 **Fig.11** shows confusion matrices for the trained classifiers to better illustrate a binary
 567 comparison between the classes in the results. The (i,j) -th element of a confusion matrix
 568 shows the (normalized) count of samples that actually belong to class i but have been
 569 classified as class j . As can be observed in this figure, a common mistake among all the
 570 models is misclassifying samples of class BS as class S. In addition, a mistake that both
 571 models is misclassifying samples of class BS as class S. In addition, a mistake that both

572 our non-linear models (DT and RF) have committed is mistaking FB samples for class PB,
 573 whereas the linear model (LR) incorrectly labeled a significant number of PS samples as
 574 PB.



575 **Fig. 11** Confusion matrix for the trained classifiers. The elements are normalized over the rows to
 576 compensate for the imbalancedness of the test data set.
 577

578 A completely different working condition of these three algorithms would lead to different
 579 results and prediction accuracy. For example, a particular formula for classifying and
 580 predicting is used for LR, whereas RF works by constructing nodes and trees. The
 581 classification model of impact droplets on hydrophobic and superhydrophobic silicone
 582 rubber surfaces obtains satisfying results on the basis of the algorithms tested on the
 583 experimental data. Linear-based algorithms (LR) are not as accurate as the more
 584 sophisticated and nonlinear algorithms (DT and RF) for classifying impact droplets in
 585 scenarios of more complex experimental conditions. This leads to inaccuracies; however,
 586 LR is slightly more effective than DT and RF at producing understandable and interpretable
 587 equations. Consequently, the strong performance of our models indicates their ability to
 588 determine the complex relationship between the various parameters affecting impact
 589 droplets.

590 3.2.1. Analyzing the Importance of Features

591 The main objectives of the classification techniques are (1) predicting the output for new
592 input features as accurately as possible and (2) providing information about the relationship
593 between the input variables and output. Some of these models are linear classification
594 models (e.g., LR) that are understandable and interpretable; however, these models may
595 not perform better than nonlinear models.

596 One of the critical inputs to these machine-learning algorithms is the feature importance
597 measurement, which can have various applications, such as reducing the number of
598 dimensions and selecting the most contributing factors in a given data set [57,58].
599 Depicting a one-to-one relationship between impact behavior and the experimental
600 parameters (including Re , We , CA , CAH , S_q , and T) is possible; however, in this
601 conventional approach, quantifying variable importance is particularly challenging in the
602 case of nonlinear relationships between parameters. Moreover, evaluating the simultaneous
603 effect of each conditioning factor on impact behavior appears impossible through
604 conventional methods.

605 We used multiclass supervised learning with a multitude of features to solve our
606 classification task. We evaluate the performance of our models when all features are
607 considered simultaneously. However, we can also isolate individual features and assess
608 their respective importance in the classification. To perform the latter, we must apply
609 distinct strategies for tree-based models (DT and RF) and logistic regression.

610 As explained above, decision trees comprise several nodes, each of which includes a
611 decision rule as a function of a single feature. Here, we define the importance of each
612 feature as the average reduction of the impurity criterion caused by that feature. Because

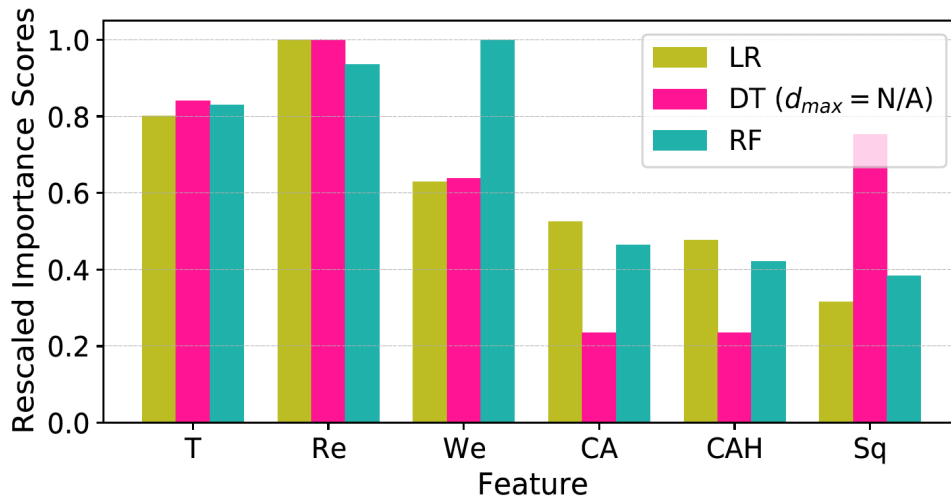
613 we use the Ginni index to measure the impurity, the resulting score is also called the Ginni
614 importance.

615 In the LR approach, we utilize the amount of change in the log-likelihood objective
616 function of the LR model that is caused by a particular feature to measure its importance.
617 More specifically, we repeat the training after removing a feature from the training data set
618 and compute the log-likelihood objective function of the resulting model (excluding the
619 regularization penalty term). The magnitude of the difference between the log-likelihood
620 of the modified and original training models is the importance score of the considered
621 feature.

622 In predicting the impact droplet process, the contribution of the different affecting
623 parameters, their effectiveness, and their influence on the accuracy of the predictive models
624 is critical. To demonstrate this, we present the relative importance of features (Re , We , CA ,
625 CAH , S_q , and T) for the three models (Fig. 12). Here, we consider the DT model without
626 any depth-dependent conditions. We observe that all the models agree that T and Re are
627 essential features to be retained. RF also assigns high importance to We number.

628 Interestingly, RF estimated that the We and Re numbers are the most important variables.
629 T is the most important feature for surface properties, followed by CA , CAH , and S_q . Our
630 experimental results (Figs. 7, 8) show that the Re and We numbers are the key factors
631 affecting droplet behavior. For example, we observed a similar pattern for all water-
632 repellent superhydrophobic silicone surfaces; however, depending on the selected Re and
633 We numbers, it provoked either full bouncing, partial bouncing, deposition, splashing,
634 prompt splashing, or bouncing-splashing. In this study, RF models perform better in

635 analyzing feature importance given their agreement with the experimental results.
 636 Moreover, looking specifically at the dynamics of impacting droplets under icing
 637 conditions, as water droplet properties and wetting properties depend on temperature, it
 638 seems difficult to say that there is a linear relationship between these factors. This issue
 639 also highlights that using nonlinear algorithms such as RF to classify droplet behaviors
 640 under more complex experimental conditions is a valid and accurate approach for
 641 predicting droplet impact dynamics.



642
 643 **Fig. 12** Importance analysis of the features; temperature (T), Reynolds number (Re), Weber number
 644 (We), contact angle (CA), contact angle hysteresis (CAH) and surface roughness (S_q) for the three
 645 trained models of logistic regression (LR), decision tree (DT), and random forest (RF)

646 **4. Conclusion**

647 Here we studied droplet impacts on hydrophobic and superhydrophobic surfaces at
 648 different temperatures and proposed design guidelines for nonwetting surfaces under
 649 droplet impingement. We applied experimental and statistical approaches to analyze the
 650 impact dynamics of water droplets at -20 , -10 , and 25 °C and discussed the influence of
 651 the substrate roughness, temperature, and wetting properties. **The experiments showed that**

652 full bouncing observed only on superhydrophobic having a $CA > 160^\circ$ and a $CAH < 2^\circ$; and
653 temperature has a minor effect on droplet dynamics on cold hydrophobic and
654 superhydrophobic silicone rubber surfaces for a wide range of We numbers and different
655 substrate temperatures. Multiple machine-learning methods were used to predict the
656 temperature-dependent droplet behavior on hydrophobic and superhydrophobic silicone
657 rubber surfaces, taking into consideration the impact velocity, droplet diameter, and surface
658 features (CA, CAH, S_q , and T). Our logistic regression-based models produced equations
659 for probability, and we combined experimental findings to model multilabel outcomes of
660 different phenomena arising during droplet impact on a solid surface. We also used both
661 linear (LR) and nonlinear (DT and RF) methods to assess the importance of surface
662 characteristics and found that We and Re numbers were the most important factors followed
663 by factors related to surface T, then CA, CAH, and S_q .

664 To the best of our knowledge, this work presents the first application of machine learning
665 to experimental results obtained for the detailed dynamic motions of a water droplet
666 impacting hydrophobic and superhydrophobic surfaces at different temperatures. Our
667 results provide a means of predicting droplet impact behavior through the application of
668 statistical LR modeling and the data-mining DT and RF modeling. All three machine-
669 learning approaches agreed well with the experimental results for classifying droplet
670 behaviors. Although all models exhibited a reasonable performance, the lower accuracy of
671 the LR model indicated that the correlation was nonlinear. The dependency of water droplet
672 properties and wetting properties on temperature makes finding a linear relationship
673 between these parameters difficult. Thus, modeling water droplet behavior on the basis of

674 these factors is not straightforward using conventional methods, illustrating the utility of
675 the machine-learning approach.

676 **CRedit authorship contribution statement**

677 **Samaneh Keshvarzi:** Conceptualization, Investigation, Methodology, Validation, Writing
678 – original draft, Writing - review & editing. **Jamshid Sourati:** Conceptualization,
679 Methodology, Software, Validation, Investigation, Writing - original draft, Writing -
680 review & editing. **Gelareh Momen:** Conceptualization, Project administration,
681 Supervision, Resources, Funding acquisition, Writing - review & editing. **Reza Jafari:**
682 Conceptualization, Project administration, Supervision, Resources, Funding acquisition,
683 Writing - review & editing.

684 **Declaration of Interests**

685 The authors declare that they have no known competing financial interests or personal
686 relationships that could have appeared to influence the work reported in this paper.

687 **Acknowledgments**

688 The authors would like to acknowledge the financial support from Natural Sciences and
689 Engineering Research Council of Canada (NSERC). Also, we would like to thank the help
690 of Mr. Vahid Asghari for machine-learning programming and Mr. Khosrow Maghsoudi
691 for sample preparation during the experiments.

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